On inclusion and sharing mathematical resources

Antonella Perucca

This contribution consists of four distinct reflections: the first two support inclusion, and the last two are a call to join forces and create (and freely share) didactic materials. Let's introduce them with questions:

- What does it mean to vary the amount of sugar in a cake, given that a baked cake cannot be undone?
- Why are female pupils so limited in their choice of role models?
- Why don't we math enthusiasts cooperate to build more free resources for teachers?
- Have you considered creating Wikipedia pages as original student projects that contribute to society?

Is it truly possible to vary quantities?

A real variable can have many facets, one of it being that it stands for a real number that can vary. Considering real-life quantities, the amount of money in a bank account can vary with time. Or the temperature can vary. It also makes sense to say, "if you vary the temperature", for example the temperature of an oven during cooking.

Speed is also a quantity that one can easily imagine varying oneself, because for example one may push or not the gas pedal (or the break) during a drive, and the speed must indeed be changed according to traffic.

A different kind of variation is "if you vary the amount of sugar" in a cake because the cake cannot be undone, so what is meant here is that with a new cake one can make a different choice and have a different amount of sugar.

In general, not everything that can be made to vary in a mathematical sense can also be made to vary in real-life. One example: the age of people in 2026 depends on their birth year, but clearly a person cannot change their birth year.

To convey the perplexity that pupils may have while been asked to vary quantities that cannot be changed by them, consider the example "if you vary the size of the dog". Indeed, imagine you are giving medicine to your dog. The amount of medicine depends on the size of the dog, however it does not help you to imagine that your dog could be different from what it is, and in any case, you cannot change the size of your dog.

We should keep in mind that certain people tend to take things very literally, or at least it takes them some effort not to do so: some of them are among our pupils. Let's consider this while using the construction "if you vary".

Choosing female role models in mathematics

Please choose your favorite mathematician as a pseudonym. Whom did you choose? I have recently asked pseudonyms to my students to be able to discuss mistakes anonymously ("name the sin but not the sinner"). Female students tended to select female mathematicians, and none of the male students selected a female mathematician.

One can find top female mathematicians as role models, old and new, for example: Emmy Noether, Maryam Mirzakhani, Katherine Johnson, and Melanie Matchett Wood. However, women in mathematics are still underrepresented and underrecognized, to the extent that it still seems difficult to have 40 % female plenary speakers at conferences (the few female mathematician superstars receive so many speaking invitations that they have to turn most of them down).

Notice the subtleties of female role models, like in the expression *Superwomen in STEM*. Do we need superpowers to be female scientists? (Or do you think Newton had superpowers?) And overly strong role models can be intimidating; the bar is too high. They should be inspiring (like the tennis player Serena Williams), but one must not think, "I need to be as good as Serena Williams to make a living out of tennis."

Coming back to the pseudonym choice, we see that we impose restrictions on ourselves that do not actually exist. We are only making a *Gedankenexperiment*, so we can be Gauss, or Thales, or Noether alike. Let's reveal the mind trap while choosing a character from

a book or a movie. For example, if you know *Pride and Prejudice*: before, I thought that the only two possible choices were Elizabeth or Jane; now, I would, without hesitation, be Mr. Darcy.

So, what's the takeaway? Make sure to also offer students female role models they can actually relate to, like a student from their school who had success in mathematical competitions, one who pursued STEM studies, or a female scientist from a nearby university. And don't forget to remind female students that they can also have male role models – why not?

Let's keep in mind that the aim of role models for female students is to convince them, on an emotional level, that they have the potential to be the best version of themselves – and, in particular, that they should not be afraid to pursue a career in STEM, regardless of their gender.

For the sake of mathematics

We care about mathematics. We do love mathematics. We would like all pupils – and why not, the general public – to care about mathematics and not hate it.

We want to support math teachers, our daily heroes, with research-based advice, original learning material, or, as math teachers, with peer support. Many of us teach future teachers, whether we are mathematicians or didacticians (and I teach both mathematics and its didactics). We all have busy agendas because, everywhere we look, there is room for improvement.

Let's exchange, cooperate, and share: when opportunities arise, let's seize them.

One VIP within GDM wishes that mathematicians take a closer look at school books. I, a mathematician, keep learning about didactics to hold better classes, to better train my Lehramtstudenten, and to better support pupils and teachers with my projects. I am on the lookout for freely available material that can be shared openly – whether it's exercises for pupils, suggestions for teachers on how to make the most of AI, or anything else.

Mathematics is one discipline, and we are many. Let's avoid a strict partition into smaller subsets; instead, let's focus on our common goals and help one another achieve them.

I wish everyone the resilience to fight long battles and the joy of making a difference. We all work hard, and we deserve our successes. As we were reminded in a spectacular talk at the GDM Tagung, "Scheitern" is a trigger to do better. In a sense, we do (and must) evolve every day.

The future holds plenty of challenges – let's face them, and let's face them together.

Save Wikipedia for mathematics education

It still looks like a miracle to me that millions of people have produced millions of encyclopaedic articles in a cooperative way: Wikipedia is a true world heritage that we take for granted but that we must also actively protect and support.

If you are not yet a Wikipedia editor, it is not difficult to become one. The English Wikipedia has something more than 7 million articles, while the German Wikipedia has something more than 3 million articles. I would guess that there are plenty of entries about mathematical objects (or mathematicians) that exist only in English. What you could then do is taking one of these entries and creating the corresponding German page. While doing so, you may enrich the content (namely, you may go beyond the mere translation or summary of the English page).

It could be that experienced users then edit parts of your page, making use of Wikipedia conventions that you are unfamiliar with. It could be that with time your page will be edited so much that you cannot recognise it anymore. Let it be. And also accept that any correction or critic is forever visible on the history of the page: this is a natural part of the contribution process (and you may choose a username which will grant you some anonymity).

Improving the Wikipedia can also mean improving the digital resources on Wikipedia Commons: if you detect a need, you can for example upload a new (or a better) mathematical image and then adding (or replacing) it in some Wikipedia page. Please opt for the CC0 licence, namely make your images public domain: this will be appreciated, e.g., by those teachers who want to use those images for their classes, as they won't need to worry about an attribution. By the way, your Wikipedia username will be linked to your images in Wikipedia Commons, in case you want proof of your contribution.

Last but not least, creating Wikipedia pages is a fantastic student project: this feeling of serving the community is a nice reward for the students' efforts.

My recommendation is to peek into all Wikipedia pages that already exist in the various languages for the entry you are considering. Indeed, we are speaking of a mathematical entry and you can often get a good idea of the content just by looking at the pictures and at the formulas. You will in particularly immediately

Example of shared material: Multiplication tables without repetitions (you have, e.g., 3 × 4 but not 4 × 3).
Free for download, language-free and without logos. Posters available in various versions. Available as mugs in two different designs: you can download and print the panorama image on a mug (as you would do for a holiday picture).
Posters, mugs, and more can be found on this page: www.antonellaperucca.net/didactics/

detect which pages are completely inspired from the corresponding English page. In any case, today you have tools to translate from all Wikipedia languages you may wish to read. The comparison of the same page in the different languages will naturally lead you to discuss the pros and cons of the various presentations. Afterwards you may come up with your own text, which ideally combines the best out of all those pages. Students may also deepen the study of the mathematical object under consideration and enrich the page content for example with references and information coming from mathematical books or different websites.

Moreover, related entries and notions can also be investigated. In a nutshell, you can keep happily busy as many students as you want for as much time as you want. I have made very good experiences, and of course I am planning to create more pages in the future whenever the occasion arises. In general, keep in mind that students, while learning, can create learning material that can be shared for the benefit of all.

Antonella Perucca, University of Luxembourg
antonella.perucca@uni.lu