Growth and success of "mathe 2000" – a privileged observer's view Laudatio anlässlich des 25jährigen Bestehens des Projekts "mathe 2000"

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It is a great pleasure and a great honor for me to represent the international community at the 22nd symposium "mathe 2000", which is devoted to the 25th anniversary of this project.

During the past three decades the international scene of mathematics education has witnessed, in various parts of the world, serious debates about the goals, the content and the methods of elementary school mathematics, which sometimes have evolved into true "math wars".

For instance, in the US there have been, since the launch of the NCTM Standards in the 1980s (National Council of Teachers of Mathematics, 1989, 2000), highly emotional debates between opponents and advocates of the reform-based approach to elementary school mathematics; between traditionalists, who still believe that the emphasis of math education should be on the direct teaching of fixed, step-by-step procedures for solving various types of math problems, and reformers, who favor a more inquiry-based approach in which pupils are exposed to real-world problems that help them develop deep conceptual understanding, number sense, reasoning and problemsolving skills, and positive affects towards mathematics. Only in 2008, the National Mathematics Advisory Panel, created by president George Bush himself, succeeded, at least to some extent, in stopping that national war (United States Department of Education, 2008).

In the Netherlands, another leading country in the international scene of mathematics education, we have seen a very similar development. Growing concern about Dutch children's mathematical proficiency in national and international assessments has led in recent years to a hot public debate about the way elementary mathematics should be taught. There were again two opposing camps: those who advocated teaching mathematics in the "traditional" manner, and those who supported realistic mathematics education, the reform-based type of mathematics education that has been conceived and further developed by Prof. Freudenthal (1983) and his colleagues and successors at the University of Utrecht (see, e.g., van den Heuvel-Panhuizen, 2001). Because of the intensity of the debate, the Dutch Royal Academy of Sciences decided to install a Committee, which wrote a report that also succeeded in calming down, at least to some extent, the public debate about the quality and future of elementary school mathematics (Koninklijke Nederlandse Akademie van Wetenschappen, 2009).

In this turbulent international context, Prof. Wittmann, Prof. Müller and the other members of the "mathe 2000" project have, during the past 25 years, worked at the development of their own approach to elementary school mathematics education, in a way that I consider quite unique and exemplarily, for three reasons that I will elaborate in a necessarily brief and superficial way in this short tribute.

View on elementary school mathematics

In terms of its view on elementary school mathematics, one of the most important general characteristics of the "mathe 2000" project is that it has, from the very beginning, refused to look at math education, and at its own position in the international scene, in extreme or polarized terms. I am aware that there exist more nuanced and sophisticated categorizations, but, I find it conceptually helpful to conceive of elementary mathematics education as a field consisting of roughly three major aspects, each of which has been central in a historically important tradition of elementary school mathematics: a mechanistic, a structuralistic, and a realistic aspect (Verschaffel, 1995).

First, elementary school math has a lot to do with memorization of basic facts, automatization of techniques for doing mental and written arithmetic, routine mastery of rules for solving standard problems dealing with number and space. Historically, this "mechanistic" element has been emphasized a lot in traditional elementary school mathematics, and it is this element that has been re-emphasized in these anti-reform movements in the US and The Netherlands that I referred to before.

Second, elementary school mathematics is about structures and patterns. In the various manifestations in concrete mathematical statements or



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Anlässlich des 25jährigen Bestehens des Projekts ,mathe 2000' wurde an der TU Dortmund eine Festveranstaltung ausgerichtet, an der 600 Lehrerinnen und Lehrer sowie Wissenschaftlerinnen und Wissenschaftler aus ganz Deutschland, ja sogar aus Belgien, den Niederlanden, Österreich und der Schweiz teilnahmen. Das Projekt wurde 1987 von den Dortmunder Emiriti Prof. Gerhard Müller und Prof. Erich Wittmann gegründet, um Konzepte und Materialien für einen aktiv-entdeckenden und fachlich fundierten Mathematik zu entwickeln, zu erforschen und zu implementieren – kein simples ,aus der Praxis für die Praxis, sondern eine empirisch und theoretisch fundierte Mathematikdidaktik mit der Zielsetzung der konkreten Unterstützung der Unterrichtspraxis.

Festredner aus Belgien, Österreich, der Schweiz und Deutschland würdigten die Arbeit des Projekts, in dem sie verdeutlichten, wie sie die Impulse von ,mathe 2000' ihre eigene Arbeit anregten und weiter entwickelt wurden. 17 Workshops am Vor- und am Nachmittag boten zudem reichhaltige Gelegenheiten, sich mit dem ,mathe 2000'-Konzepten und Materialien eigenständig auseinander zu setzen. Aus Anlass des 25jährigen Bestehens des Projekts erschien zudem das Buch ,Zahlen, Muster und Strukturen', welches von den Projektleitern Müller, Selter und Wittmann herausgegeben wurde und sowohl an theoretischen Grundlagenaufsätzen als auch anhand von vielen konkreten Praxisberichten aufzeigt, wie im Unterricht Spielräume für aktives, verstehensorientiertes Mathematiklernen und beziehungsreiches Üben geschaffen werden können.

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problems, there may be a common principle, a common pattern or structure, an underlying "big idea", that has to be discovered, explored, understood, expressed, formalized, generalized ..., by the learner, and that should become part of his or her conceptual toolbox. This aspect was central in the structuralistic approaches to elementary school math, such as the New Math movement, that was dominant in the fifties to eighties of the previous century in various parts of the Western world, but is also emphasized in current approaches that emphasize, for instance, the role of pre-algebra in elementary school mathematics.

Third, mathematics is a human problem solving activity; it is about establishing links between real world situations and mathematics, in both directions; it is about seeing the mathematics in the real world and about using mathematics to make sense of this world, to understand and manipulate it, with a view to efficiently solve problems that arise in that world. This aspect of "mathematical modeling and applications" is prominently present in approaches, such as the Dutch realistic approach to mathematics education (although it would be too simple to reduce RME to that aspect).

Just as in the world-famous tale of a group of blind men each touching a part of the elephant to learn what it is like, but every single man being unable to get a complete picture of what it essentially is, each of these three aspects point to a truly essential feature of elementary school mathematics, but does not tell the whole story of what it is about. The great value of the "mathe 2000" approach is that it departs from a view of elementary mathematics education that integrates in a well-balanced way all three aspects. It does so both in its theoretical foundations and in the concrete textbook pages and materials of its textbook, Das Zahlenbuch. To the best of my knowledge, there are few textbooks in the world that have been so successful in realizing this balance so subtly and so successfully as Das Zahlenbuch.

Moreover, in realizing that subtle balance between these three major pillars of elementary school mathematics, it adheres to three principles that have been found in the curricula of the world's highest-performing countries, according to a recent study by Houang and Schmidt (2012) namely (1) coherence (the logical structure that guides students from basic to more advanced material in a systematic way); (2) focus (the push for mastery of a few key concepts at each grade rather than skim over dozens of disconnected topics every year); and (3) rigor (the level of difficulty at each grade level).

Methodological approach

Closely related to the above-mentioned international debate between advocates and opponents of reform-based approaches to math education, there is an ongoing methodological fight about the kind of scientific research that is primarily suited and needed for improving elementary school mathematics. Stated again somewhat boldly, there are, on the one hand, researchers who adhere the socalled "evidence-based approach", which postulates that only effective type of research is the (quasi-)experiment, whereby one compares the effect on learners of two or more approaches to teach a given mathematical topic, with randomly selected classes, in well-controlled conditions, using only psychometrically adequate standard achievement tests; and, on the other hand, those who argue that this evidence-based approach is not and will never be able to capture the rich, complex and contextual nature of teaching and learning in a real mathematics classroom, and therefore argue that the only useful kind of research is of a more qualitative nature, that documents in detail how one arrived at the design of a new teaching/learning unit, how teachers and learners reacted to it, and what was learnt from it in view of the improvement of the design of that unit (Verschaffel, 2009). Also in this international methodological battlefield, the "mathe 20000" project has always taken a nuanced, broad-spectrum view, by pleading, on the one hand, for the existence of "design experiments" as a central research method in the domain of mathematics education, but, on the other hand, also supporting more large-scale and systematic evaluation studies aimed at unraveling the relative strength and weaknesses of its newly designed instructional materials and approaches. As illustrations of the former, I refer to Prof. Wittman's paper "Mathematics education as a design science", published in Educational Studies in Mathematics (1995), which has become an internationally recognized "classic" in the field of mathematics education, as well as Prof. Selter's exemplary design study about building on children's mathematical productions in grade 3, published in 1998 in the same journal. Illustrations of the latter are the evaluation studies by Moser Opitz (2002) and Hess (2003), both comparing teaching and learning in classes in which a traditional textbook was used with teaching and learning in classes which worked with (an adaptation of) the *Zahlenbuch*, and both providing substantial empirical support for the "mathe 2000" approach, particularly for the mathematically weaker children.

Role of the teacher

Referring back to the two reports that tried to stop the math wars in the US and The Netherlands, it is interesting to see that according to both reports the key to improving children's mathematical proficiency does not lie in the textbook in itself, but in the competencies of the teachers who have to use it. And, by these competencies, they do not only mean their mathematical content knowledge, but also, and according to some even primarily, their "Fachdidaktische Kompetenz", or, in Shulman's (1986, 1987) terminology, their pedagogical content knowledge (PCK). Many studies and surveys have indicated the importance of this PCK. In a recent German study (COACTIV project - see, Baumert et al., 2010), it has been shown that students taught by teachers with a high PCK showed better PISA results than those of other students, mainly because teachers with a high PCK design their teaching so that the students are more actively cognitively engaged. Further analyses revealed that PCK has greater predictive power for student progress and is more decisive for the quality of instruction than their content knowledge (Baumert et al., 2010, p. 164). Moreover, the available international research on mathematics teachers' knowledge and professional development (as nicely summarized in a recent publication by the Education Committee of the European Mathematical Society (2012) headed by prof. Konrad Krainer), indicates the positive impact of "collaboration" among teachers and of teachers' collegial learning, i.e. of teachers belonging to "communities" consisting of experts, teachers and researchers and improving their teaching actions and upgrading their professional theory through unfolding their learning process in cooperation with the other members of the community. Clearly, the "mathe 2000" project has, from the very beginning, deeply endorsed the idea that the teacher is the critical factor in the curriculum implementation process, and that, therefore, a textbook series project without a parallel well-established supportive system for its teachers, is doomed to fail. This is not only evidenced by the two excellent volumes of the Handbuch produktiver Rechenübungen (Wittmann & Müller, 2000-2002) that accompany the textbook Das Zahlenbuch, and that provide the teachers with the PCK and the accompanying beliefs needed to implement the textbook in a proper way; but also by the organization of the annual meetings of the "mathe 20000" community allowing intensive exchanges of ideas, findings and experiences between teachers, researchers and other kinds of experts.

As a scholar from abroad, it was a great privilege to observe from close-by, through my longstanding and intensive contacts with the members from the Dortmund "Institut für Entwicklung und Erforschung des Mathematikunterrichts" (IEEM), the development of the "mathe 20000" project. The project can be really proud of what it has accomplished during the past 25 years and the impact it has had on the research on and practice of elementary school mathematics, in Nordrhein-Westfalen, in Germany, and abroad. I wish you all very nice and stimulating conference celebrating this 25th anniversary.

References

- Baumert, J., Kunter, M., Blum, W., Brunner, M., Voss, T., Jordan, A., Klusmann, U., Krauss, S., Neubrand, M., & Tsai, Y.-M. (2010). Teachers' mathematical knowledge, cognitive activation in the classroom, and student progress. *American Educational Research Journal*, 47, 133–180.
- De Corte, E. & Verschaffel, L. (2006). Mathematical thinking and learning. In Damon, W., Lerner, R., Sigel, I & Renninger, A. (eds.) *Handbook of child psychology. V. 4: Child psychology in practice*, pp. 103–152. New York: Wiley.
- Education Committee of the European Mathematical Society (2012). It is necessary that teachers are mathematically proficient, but is it sufficient? Solid findings in mathematics education on teacher knowledge. *Newsletter of the European Mathematical Society*, March 2012, 46–50.
- Freudenthal, H. (1983). *Didactical phenomenology of mathematical structures*. Dordrecht, The Netherlands: Reidel.
- Hess, K. (2003). Lehren zwischen Belehrung und Lernbegleitung. Einstellungen, Umsetzungen und Wirkungen im mathematischen Anfangsunterricht. Bern: h.e.p. Verlag.
- Koninklijke Nederlandse Akademie van Wetenschappen (2009). Rekenonderwijs op de basisschool. Analyse en sleutels tot verbetering. Amsterdam: KNAW.
- Moser Opitz, E. (2002). Zählen, Zahlbegriff, Rechnen. Theoretische Grundlagen und eine empirische Untersuchung zum mathematischen Erstunterricht in Sonderschulklassen. 2. Auflage. Bern: Verlag Paul Haupt.
- National Council of Teachers of Mathematics. (1989). *Curriculum and evaluation standards for school mathematics*. Reston, VA: National Council of Teachers of Mathemetics.

- National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA: National Council of Teachers of Mathematics.
- Schmidt, W. (2012). Seizing the Moment for Mathematics. Education Week [American Education's Newspaper of Record], Wednesday, July 18, 2012, Volume 31, Issue 36, pp 24– 25. See http://www.edweek.org/ew/articles/2012/07/ 18/36schmidt.h31.html?cmp=ENL-EU-SUBCNT
- Selter, C. (1998). Building on children's mathematics A teaching experiment in grade 3. Educational Studies in Mathematics, 36, 1–27.
- Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher*, 15, 4–14.
- Shulman, L. S. (1987). Knowledge and teaching: Foundations of the new reform. *Harvard Educational Review*, 57, 1–21.
- United States Department of Education (2008). Foundations for success. The final report of the National Mathematics Advisory Panel. (Retrieved January 17 2009 from http://www.ed. gov/about/bdscomm/list/mathpanel/index.html)
- van den Heuvel-Panhuizen, M. (Ed.) (2001). *Children learn mathematics*. Utrecht, The Netherlands: Freudenthal Institute, University of Utrecht.
- Verschaffel, L. (1995). Ontwikkelingen in de opvattingen over en de praktijk van het reken/wiskundeonderwijs op de basisschool. In: L. Verschaffel & E. De Corte (Red.), Naar een nieuwe reken/wiskundedidactiek voor de basisschool en de basiseducatie. Deel 1. Achtergronden (pp. 95–128). Brussel: Studiecentrum voor Open Hoger Afstandsonderwijs (StOHO).
- Verschaffel, L. (2009). 'Over het muurtje kijken': Achtergrond, inhoud en receptie van het Final Report van het 'National Mathematics Advisory Panel' in de U.S. Panama-Post – Reken-wiskundeonderwijs: Onderzoek, ontwikkeling, praktijk, 28(1), 3–20).
- Verschaffel, L., & Greer, B. (in press). Domain-specific strategies and models: Mathematics education. In Spector, J. M., Merrill, M. D., Elen, J. & Bishop, M. J. (eds.) Handbook of research on educational communications and technology. 4th ed. New York: Springer Academic.
- Wittmann, E. Ch. (1995). Mathematics education as a design science. *Educational Studies in Mathematics*, 29, 355–374.
- Wittmann, E. Ch., & Müller, G. R. (2000–2002). Handbuch produktiver Rechenübungen (Bd. 1, Vom Einspluseins zum Einmaleins, und, Bd. 2, Vom halbschriftlichen und schriftlichen Rechnen: Vom halbschriftlichen zum schriftlichen Rechnen). Stuttgart: Ernst Klett Schulbuchverlag.

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