wenn man mit Vorsicht und nach Diskussion vorgeht, kann sich die Arbeit doch lohnen – und das selbst, wenn am Ende nichts geändert wird. Dass dann aber zumindest einige Lehrkräfte sich wieder bewusst gemacht haben, wo Stolperfallen lauern – das wird dem Unterricht gut tun.

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The Challenges, Reforms, and Future Prospects of Elementary and Lower Secondary Mathematics Education in Germany

Michael Neubrand

Im Oktober 2016 fand an der East China Normal University (ECNU) in Shanghai der dritte sogenannte „Chinesisch–deutsche Didaktik-Dialog" statt (Mentoren auf deutscher Seite: Dietrich Benner und Hilbert Meyer). Die Gespräche machten vor allem das überraschend reichhaltige Nachdenken über die Allgemeine Didaktik in China sichtbar und zeigten die vielfältigen Verbindungen zu deutschen Didaktik-Traditionen auf. Den vierten chinesisch-deutschen Dialog, nun mit deutlicherem Bezug zu den Fachdidaktiken, richtete Ende Mai dieses Jahres das IPN in Kiel aus.

Die ECNU ist für ihre internationale Ausrichtung in pädagogischen Fragen bekannt. Für die Mathematikdidaktik wird das etwa dadurch sichtbar, dass der auf Hamburg folgende ICMI im Jahr 2020 in Shanghai sein wird (Local Organizer: Binyan Xu). Man interessiert sich in Shanghai also von jeher für Entwicklungen in den Schulen weltweit. Es wird dort dafür seit Jahrzehnten die internationale Zeitschrift *Global Education* herausgegeben. Für diese Zeitschrift habe ich am Rande des Didaktik-Dialogs ein ausführliches Interview gegeben. Dieses wurde von der MA-Kandidatin Yamei Ke, die am „Institute of Curriculum and Instruction" der ECNU arbeitet, geführt. Ziel des Interviews war es, für die fachlich nicht spezialisierte Leserschaft der Zeitschrift in groben Linien ein Bild vom Zustand und von den Problemen des deutschen Mathematikunterrichts zu zeichnen. Die ausführliche Konversation mit Ke Yamei zeigte mir, dass sowohl Basisinformationen (der erste Teil mit Zielen, Inhalten und Entwicklungslinien) als auch Problemaufrisse (der zweite Teil in Form der Benennung von fünf „core problems") nötig waren.

Das Interview gab mir die Gelegenheit, unser eigenes Feld gewissermaßen „selbst von weit weg" und durchaus spontan reagierend darzustellen. Gerade weil dann auch persönliche Sichtweisen und Einschätzungen zu Tage treten, erscheint mir ein solcher distanzierter Überblick durchaus lohnend auch für Leserinnen und Leser aus unserer Community. Dies ist der Grund, warum dieses Interview nun hier in den *Mitteilungen der GDM* abermals erscheint. Dafür habe ich die ursprüngliche Version in englischer Sprache, in der sich Yamei Ke und ich verständigten, beibehalten. Das Interview ist ca. ein Jahr nach dem Treffen in chinesischer Sprache von Yamei Ke veröffentlicht worden: *Global Education*, vol. 46 (11), pp. 3–11, November 2017. *Global Education* hat zugestimmt, den englischen Text hier für die GDM-Mitglieder erneut zugänglich zu machen.

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Abstract. Keeping its original form of an interview, this article presents a discussion about the challenges, reforms, and the prospects of mathematics education in Germany. The interview addresses aims and goals, contents and processes of mathematics teaching. Compared with the guiding ideas some years ago, more emphasis is put on modeling today. The idea that every student should have enough mathematics knowledge and the disappointing results of Germany in the PISA-2000 comparison may have caused this change. Moreover, Germany faces, like other countries do, some basic problems in mathematics teaching, addressed here as the balance problem, the coherence problem, the curriculum problem, the classroom organization

problem, the computer problem, and the teacher education and development problem. The interview also tries to show how these problems are going to be tackled in Germany. For the prospects, being able to apply mathematics in the real world, being able to understand the mathematical concepts and rules, and having the potential of cultivating the own thinking are the main concerns of mathematical literacy in Germany.

1 The Guiding Ideas of Elementary and Lower Secondary Mathematics Education in Germany in the New Century

Ke Yamei: What are the guiding ideas of elementary mathematics educations in Germany today?

Michael Neubrand: To discuss the main ideas, I'm going to consider three aspects: the aims and goals aspect, the content aspect, and the processes aspect.

The first aspect towards our question is thinking about the aims and goals of teaching mathematics in schools. There are two parts. One is to empower people to be able to do mathematical things in daily life. Like calculating in the market, looking on a graphic to figure out what it can tell, orientating in the space, and so on. Maybe this is not yet complete mathematics, it is rather more a kind of brief mathematics which is only the beginning. The second part is introducing mathematical thinking. Mathematical thinking has above all to do with seeing more than just calculation, such as seeing the structure, seeing the patterns, seeing the generalities, seeing the concepts, and so on. Daily life and introduction to mathematics are the two parts of the aims and goals of mathematics.

The second aspect is thinking about the content. What topics should be dealt with in school? This aspect contains four main parts, numbers & measurement, functions & equations, geometry, and statistics & probability.

In Germany, we start with numbers. We have a quite strict procedure to introduce into numbers from Grade 1 to Grade 9. In Grade 1, we only deal with the whole numbers 1 to 20, learning addition and subtraction. Both kinds of goals, as mentioned before, should appear: being fluent and understanding the structure. For instance, students should be able responding quickly that 5 plus 6 equals 11, but on the other hand, they should recognize and understand that the same result 11 also comes from 4 plus 7, 3 plus 8, and so on, by systematically changing the summands. Even in Grade 1, we have these twofold aims. Then in Grade 2, we proceed to 100, but again, being fluent and understanding the structures, e.g. the decimal partition, are both at the agenda. At the end of Grade 2 and the beginning of Grade 3, it comes into multiplication, students learn 4 times 3 is 12 and things like that, we call it the "one-times-one". Again, we get the same aims and goals, being fluent and seeing the structure. There are lots of mathematical patterns. In Grade 3, we go further up to 1000; in Grade 4 it goes up to a million, but still only talking about the whole numbers. In Grade 4, we also have the multiplication with more complicated things, like 84 time 33, you have to write down and calculate quickly. In Grade 5 and 6 we introduce the fractions, and in Grade 7 the negative rational numbers. In these Grades, both aims are important, too. The aim at being fluent means you should be able to calculate quickly, using some calculation rules. The other aim is seeing the structure, like using the number line; you should judge where the number is on the number line, and what the operations mean. In Grade 8, we start things that go beyond fractions, e.g. radical expressions like $\sqrt{2}$. In Grade 9, we have the number π.

One quite important aspect of numbers is that they are often and throughout the curriculum used to express measurable quantities, equally in daily life, technology, science; e.g. by numbers one measures money, distances, volumes, electricity, etc. The functions aspect we are talking about next is closely connected to dealing with numbers in that way.

The second part of the content aspect is functions and equations. This is more abstract than numbers. It already starts in the lower Grades. Even students in Grade 2 should be able to calculate an equation like 5 times which number equals to 20. Above all, the structure aspect comes in: students should have an idea of the pattern. The thinking in patterns, relations and sequences comes out early. It then, from Grade 7 on, should be turned into the concept of functions. In Grade 7, we introduce variables to express an unknown number, but still being able of doing calculations with it. This is the basic idea in Grade 7 starting with linear equations. In Grades 8 and 9, we teach quadratic equations and the resp. algorithm to calculate the solutions. There are multiple structure aspects, e.g. the graph is no longer a line, but it takes the form of a parabola, or one equation can have two solutions.

The third part of the content is geometry which means the dealing with forms, shapes, and visualization. These ideas are also going through the whole education system. We start in Grade 1 with looking at easy patterns. For instance, general forms like a square, a circle, or a cube are considered. In Grade 3, we calculate the area of rectangles. And then it becomes more and more complicated, and it proceeds to the classical geometry. In Grades

7 and 8, we start with the elements of geometry, lines and triangles. A triangle has angles, an angle has a bisector, and further things like that. In Grades 8 and 9, we learn further things, like that you can draw a circle around a triangle, how you should calculate an area, and so on.

The fourth content part is statistics and probability. What is the chance of an event? In Grades 1 and 2, already, we have some easy ideas on what is probability: Is it equally probably to get a "6" or a "1" when rolling the dice, contrary to the intuition? In higher Grades, we start to learn some more complicated concepts around probabilities and statistics. A classical task is: A class is going to the sport fields to exercise long jump. The student A jumps 2.5 m, the student B does 3.2 m, and so on. Then another class does the same. The one class has 28 students and the other one 23 students. Which class is the better? Students learn to use the mean number or other statistical indicators in Grades 7 to 9.

The third aspect of the main ideas is the processes aspect. This aspect addresses the main processes when doing mathematics. I divide it into six parts. The first process, and the one which is mostly associated with mathematics, is that mathematics contains algorithms, i.e. the option to follow a predefined rule to get a result. A second mathematical process is argumentation: Students should think logically which finally turns into proofs. In Grade 9, a student must be able to proof a simple theorem, especially in geometry. This second process should always stand beside the first one. They are equally important since argumentation also needs rules. The third process is communication, i.e. being able to talk to your neighbor, talk to and listen to the teacher, reading the book, or writing something down. You can communicate with someone else, sharing your opinions and explaining your calculations. Communication is quite close, but not the same as argumentation. Saying what is behind the mathematical things is not possible without the process of argumentation. The fourth central process in mathematics is representation, i.e. being able to characterize a certain mathematical rule or concept in various forms, and equally to change these representations according to the resp. needs. For instance, here is a function, you should know the representation of that function as a graph, or as a table, or as a formula, and still see it is the same concept being represented differently. We can start this process very early. In Grade 1, you should know that three dots on the paper may mean the number 'three', and three apples or a triangle could also designate 'three'. The fifth part of the processes aspect is being critical. What does it mean? What do I know if I know that? Is that always true? How is that used? Things like that. Especially in statistics, you should be able to detect fakes, and generally, the task to find out if there is a logical mistake must be ubiquitous. The sixth important process is modeling and problem solving. The modeling process is about seeing a certain situation in the real world, and to cope with it by mathematical means. The problem-solving process it is about to see a certain situation in the mathematics and solve this problem. Modeling and problem solving are central mathematical processes since they are to combine all the aforementioned processes into an overall mathematical practice.

Ke Yamei: What are the differences between these ideas compared with the guiding ideas of elementary mathematics educations in Germany in the past? Has there anything changed?

Michael Neubrand: Around 1990, we surely didn't have such strong emphasis on modeling, as it is today. This is mainly what has changed in the so-called 'Bildungsstandards' (Principles and Standards for teaching mathematics) since 2004. 'Bildungsstandards' are an orientation frame, set by the Federal Administration, for teaching mathematics in schools. From the political viewpoint, the 'Bildungsstandards' were a new development, since they are equally valid in all 16 States of Germany, while educational and cultural affairs are still under the accountability of the single State.

This change was not without the objection of some mathematics educators and teachers. Some did not attend to that strong movement into the application field. Nevertheless, the 'Bildungsstandards' are in the field now, and of course they contain more than just the focus on modeling, e.g. they highlight the important role of the mathematical processes and practices. Thus, the only orientation to the content in former syllabuses was supplemented by the processes aspects, as we discussed it before.

However, there was also some change in the ideas of the teaching processes in the class. To grasp the full picture, we must therefore think about the whole course of action from setting up the education guidelines towards the teaching in the classes. We do it in the next paragraph by pointing to various central problems in that complex field.

Ke Yamei: What caused these changes?

Michael Neubrand: There are inner factors and outer factors. The idea that every student should have enough mathematical knowledge is one of the inner reasons for this change. 'Every student' means that there have to be some connections to the everyday life. It means to open the mathematics in school to all students and make it interesting for all.

The outside reasons mainly came from PISA, the even in China well known "Programme for International Student Assessment" of the Organization for Economic Co-operation and Development (OECD). In Germany, PISA-2000 changed a lot; some people even called it the 'PISA shock'. All the newspapers were full of it since PISA-2000 reported that Germany was below the international average in mathematics. But the German government and the German population could not believe that, recalling that Germany had such a rich tradition in education, pedagogy, and the philosophy of education; thus, so the public opinion, Germany couldn't be among the lower achieving countries in the world. It was really a shock. After this shock a lot of initiatives came out to renew mathematics education, reading education, science education, and so on. These projects finally led to the 'Bildungsstandards'.

2 The Challenges of Elementary and Lower Secondary Mathematics Teaching in Germany

Ke Yamei: Do you think there are any problems in real mathematics teaching today?

Michael Neubrand: I think the best way to answer this very complex question is by pointing out several core problems of mathematics teaching (and of other subjects, respectively) – surely, in such a general perspective these problems do not only exist in Germany.

I distinguish five of these core problems. All these problems are dialectic, in a way. They always attend to show a certain tension between two, and sometimes even more, poles. The great challenge for the teacher, and by the way also for the researcher, is to position a teaching unit, an event in the classroom, the short-term planning of the teaching in the next lesson, as well as the long-term construction of the curriculum, even oneself as a person engaged in teaching and learning, somewhere within that tensions. One always should recognize the conditions of the individual, classroom-bounded, social and political circumstances.

First, I think a characteristic tension exists in mathematics itself; I call it the *balance problem*. I start once more with the already mentioned aims and goals, i.e. fluency vs. conceptual understanding. The central problem is to find out how to have the right balance between the two poles. Some teachers believe it is good to understand the concepts, some point to the necessity of commanding the standard algorithms. But if you neglect being fluent, then it is hard to understand the concepts, and vice versa, no wise control of an algorithm is possible without seeing the conceptual sources behind. So, the question is how to balance or how to combine these two sides. In my opinion, the actual problem is that too many teachers seem to think the students can understand without having factual knowledge and without being able to do calculations quickly. But, one cannot understand without having enough knowledge, and one cannot have knowledge without understanding. There must always be a balance. That is what I mean by the balance problem.

The second problem, as I would say, is the *coherence problem*. By coherence I mean what the students have learned in Grade 1 or 2 should appear again in Grades 7, 8 or 9, and what they learn in, say, geometry has influence on, say, algebra. But for the students this is often not self-evident. Therefore, it is a matter of teaching to connect the things; but it is a challenge for the teachers. The coherence problem obligates teachers and students to see the mathematical issues as connected, and teaching and learning them as they are connected. The coherence problem has two sides: It is about teaching the students how a certain matter is connected, and for the students becoming conscious about the connections. There is always a danger that students learn mathematics just as a stuff of this one lesson, and that this only one lesson has nothing to do with the next. However, mathematics is a subject covering long distances where one concept is coherently related to another and another. You must keep in mind that mathematics has long strings of concepts strongly tied together. For instance, take the numbers: We have the natural numbers $1, 2, 3, \dots$ in the primary school, and we have the fractional numbers in grade 6, and we have the real numbers like π in Grades 9 or 10. These all are numbers and there is a coherence of the number concept from the early primary level up to the senior high school. I think that teachers have to be aware of the necessity to have this coherence, drawing all aspects into consideration, the epistemological, the historical, and the didactical perspectives including the changes these concepts will go through. This is meant when I make a plea to keep open the coherence problem.

The third core problem is the *curriculum problem*. This problem puts questions on the level of the content of mathematics going to be taught in schools. In Germany, the contents remained more or less unchanged over the last 30 years. But the circumstances have changed, i.e. the time devoted to mathematics, the diversification of the schools, the electronic devices that are now available (see later), the new fields of application, etc. So, we must make something like an update. What is needed and what is not needed so that we still have coherence and not having too many diacritic topics which disturb the students? Take as an example the problem that we have fewer lessons in mathematics than before. In most secondary schools, we often have mathematics lessons only three hours a week. That is not so much. So, it is necessary to choose which content is important and which one is not so important. What should we omit, in a way that the rest still gives a coherent picture of mathematics? One cannot just cut out, but one is forced to define a coherent curriculum. This is the curriculum problem. It is a problem of the administration; however, we as mathematics educators are in heavy duty for it, too.

The fourth problem is the *classroom organization problem*, i.e. the problem of choosing the appropriate social context in the class. I think of that problem as challenging the ways how the learning environment in the classroom is going to be organized. How to convince the teachers that besides the (still valuable under certain circumstances) teacher centered ways, there should be other forms like student-centered learning, self-directed learning, learning in life-situations, etc.? How can one do that especially in mathematics? This problem addresses again a kind of balance. The methods problem is to decide which method is suitable for which content. There is not the one method which suits any kind of content. It is up to the teacher to decide from a bundle of diverse methods which method is adequate for this or that content. Teachers should be able to argue about and choose the best method according to the situation, and they should be aware that the classroom's social organization cannot be discussed without discussing the deeper roots of the content as well. We come back towards that problem with more examples and considerations at the end of this section of the interview.

The fifth problem is the *computer problem*. This problem is about using the computers and all other modern devices in mathematics teaching in a fruitful and sense-making way. It is a modern problem, and again, it forces both knowledge and understanding. Doing things with the computer should make sense. It is not just computer for fun. I think this problem is universal, geographical and in the time dimension. In China you have the same problem, and every three years or so we have the new computer problem with the then new devices. Thus, once more, computers must make sense with respect to the mathematics, not just make things easier. To incorporate computers into the ordinary mathematics so that the mathematics can benefit from the computer (and even adjust itself): This is the computer problem.

Finally, the sixth problem is the *teacher education and development problem*. This is really a big problem. It has two sides. On the one hand, teacher education at the universities is always a matter of discussion. The actual kernel of that discussion is how to incorporate more practice of teaching into the studies at the university. But that only makes sense if the practical experiences of the students are accompanied by theoretical reflections. The three poles for weighing out are the subject mathematics, the basic dimensions of mathematics education as a scientific discipline, and the practice in the classroom. The other side of this problem is the further professional development of the teachers. In Germany, professional development is not compulsory for teachers, and thus, we have too less offers for further education of teachers. Meanwhile, things change, as we now have a new German-wide Center for Professional Development of Teachers of Mathematics; but still the organizational issues are with the 16 States. However, the essence of the problem is that teachers have to recognize professional development as a part of their professional life. By the way, I have learnt in China that it is not only possible but will be really taken that a teacher works some years in school, and then wishes to go back to the university for additional studies, to come back to school after, say, one year. I wish that this could also be viable in Germany.

Ke Yamei: What measures have been taken in view of the above-mentioned problems in Germany?

Michael Neubrand: Many of the problems we discussed before can only be solved by fostering the discussions among teachers, mathematics educators, and other stakeholders. There should always be a broad discussion, even a societal debate, when the aims and goals of teaching mathematics are affected. But this takes time and deliberation from all sides.

E.g. to solve the balance problem, first of all the teachers have to become aware of the problem. They have to see the problem, e.g. in further education courses. However, it requires a sound conceptual knowledge of the various issues we discuss in the mathematics education lectures at the university. The same holds for the coherence problem. E.g., a topic in professional development courses could be how to write, facing that problem, curriculum and teaching plans, writing a teaching booklet and things like that. To solve the computer problem, we also need further education, letting aside the availability of software as a financial problem of some schools. All these actions, however, depend on being conscious of the problems by all stakeholders, by the teachers, the teacher educators, and the public institutions that are concerned with schools.

The curriculum content problem is forehand a matter of the administration in each of the 16 States. But the administration has the commission to discuss it very carefully, esp. with the mathematics education community; I'm sure, it is a long discussion process, including societal debates.

To solve the social organization problem in the classrooms, we gradually try to change the teaching methods. However, one then has to be aware that traditions will be questioned, and this is not an easy mission. Previously, we had one predominant method of teaching mathematics, the question-and-answer method, which you also find in Chinese schools: The teacher asks a question and the students try, or even only guess, to answer it as they think it is in the teacher's sense. Now, we try to open the field for different methods, not only to be used, as before, for the reproductive parts of the mathematics lessons, i.e. for exercising or memorizing, but in specific ways also for those parts of the lessons which are devoted to the detection and elaboration of new concepts.

For instance, we have the working in groups of a few students, independent work of the single student, each without the teacher's help and instruction but receiving assistance and coaching. Furthermore, we know project work when the teacher gives a very complex task with many aspects; the students then should work on it by themselves (or in groups) independently, e.g. go to the internet to figure out the data, correcting the data, ordering the data, thinking how to present the data, and how to communicate the data when they are going to report about it in class (or even in wider contexts). Sometimes we also have work outside the school. For example, say once a year, some teachers will visit a mathematics museum, like the Science Museum here in Shanghai, with their students. There are also other methods like individual teaching: Different students can learn at different speeds and trajectories while the teacher gives them different and rich materials. Homework has changed, too. In former times, homework was just repetition. Nowadays, there are many kinds of homework. For instance, data collecting, figuring out wider connections, making a drawing which is too complex to do in the classroom, and things like that.

One should, however, point out that all these teaching methods cannot be discussed nor can they be brought to a final decision in a lesson without considering the various possibilities the content is endowed with mathematically and didactically. Sometimes it depends on the ways how specific contents are going to be arranged if a certain teaching method can be applied. A key variable in mathematics teaching are the tasks given to the students, and above all, their mathematical and didactical potential. Teachers trigger and control what happens in the mathematics classroom by the tasks they select and construct, and how they put them into the work of the students. We have a lot of empirical evidences for those mechanisms.

The teacher education and development problem should be at the agenda of the administration, but it is a task for the whole society. One has to think about how to prepare the teachers for teaching. Teachers need a sound content knowledge (CK). It is a question for the university to adjust the level, not too high and not too flat. Then, the teacher needs pedagogical content knowledge (PCK). We must therefore have a certain amount of time in teacher education programs, which is devoted to the field of mathematics education. As a third component, pedagogical knowledge (PK) must find the adequate place. How to arrange these three components, CK, PCK and PK, can only be decided after a broad societal dialogue, from students, teachers and parents up to the universities and ministries. And we have the transformation of these knowledge-based components into the acting in the classroom; this is a question of its own, sticking again to the necessity of having some reflected practice in the teacher education courses.

The adequate way to solve the in-service teacher education problem is to convince the teachers. However, it is up to the mathematics education community to give enough ideas being offered to the teachers and being sufficiently close to the teachers' needs and expectations.

Ke Yamei: Have you ever thought about making the in-service teacher education compulsory?

Michael Neubrand: Yes, some people think about it and some administrators think about it, too. But the problem is that things will not necessarily become better then. It is a work like changing a system. If you want to change a system, you must let the system change and develop it from inside. Only if more and more teachers agree to go this way, change is possible. So, a slight pressure from outside could be useful, but still, you have to convince the teachers.

3 The Future Prospects of Elementary Mathematics Education in Germany

Ke Yamei: Does Germany have any new thoughts or prospects for mathematics education?

Michael Neubrand: Remember the six problems we discussed in the central part of this interview. These are, to me, the essential problems. More or less, they will stay as open problems. They won't just be solved one day; these are fields of problems we'll always encounter in mathematics education, as well as in other areas of teaching. Moreover, these aren't isolated problems, thus that there can't be solved the one problem, and then tackle the next. Thus, we can hardly say we have completely new thoughts and directions.

But if you think of the 'Bildungsstandards' we already mentioned, we have a starting point. The general thinking in mathematics education today has at least some landmarks: The one is not to forget the mathematical practices and processes in the classroom: It's not only the content that has to be passed on, but also attitudes towards mathematics and reflections what it could mean to do mathematics (see what we discussed at the beginning of the interview but be aware of what we called the "Balance Problem"). The second landmark, of a quite different quality, could be that we are looking more than in earlier years to the outcomes of the mathematical education in the schools. We now have a countrywide monitoring system to see the progress. However, even here we encounter the "dialectic" we spoke about throughout this interview: Monitoring could be fruitful but bears the danger of "teaching to the test" always in it and maybe also some overburdening of teachers and students by regularly measuring the progress.

Ke Yamei: What is the understanding of mathematics core literacy in Germany?

Michael Neubrand: With this question, we come back to the beginning of the interview. What we discussed there as the guiding ideas of mathematics teaching in Germany can be summarized towards a description of mathematical literacy. Mathematical literacy, as we see it in Germany, has three fundamental and interdependent aspects:

The first aspect is being able to apply mathematics in the real world by modeling. However, modeling is more than just solving everyday problems by standard calculations. With respect to the literacy idea, it should enclose to realize that mathematical models serve different purposes. In connection to the real world, say when dealing with a technical problem, mathematical models aim at describing the structure of the problem, to detect the critical parameters, to understand why something happens or fails. These considerations all go beyond the sheer solution but point to the general nature of a mathematical model. It is therefore not surprising that mathematical models can and should as far as possible also serve as the origins of mathematical concepts, and, possibly, the origins of critical thinking.

The second aspect is being able to understand that mathematics is a subject of its own nature, with its own rules and methods, its own language, its own sense coming from inside. It must also be a part of mathematical literacy to understand this world in its own, at least parts of it. This aspect of mathematical literacy stretches from the ability of doing calculations (remember the keyword "fluency" we more than once encountered) to some insight into proving as one of the decisive and characteristic methods in mathematics.

The third aspect of mathematical literacy is that learning mathematics in school should also have the potential to cultivate the students' own thinking. It should reach out to all fields of intellectual behavior. Mathematics is, also, about to learn thinking. Again, as so often before, this is an ambiguous claim: Mathematics can be a field in which one can learn the rules of thinking, but the transfer of these rules into other fields is not obvious at all. Whatever you think, it must be logical, clear, ordered, cultivated, reflected, but anyway the responsibility of your thoughts is still on you and depends on the situation the problem is embedded. Thus, mathematical literacy has a person in mind that is able to think independently.

These are three facets of what mathematics literacy should be in the German understanding. There is a big consent in Germany about these three main aspects of mathematical literacy, but the aspects are interconnected and should be seen as a whole.

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