International Review

New AIR Study Compares the Quality of U.S. Math Instruction with Singapore, a Recognized World Leader

U.S. Trails, But Both Nations Could Learn from Each Other[1]

A study by the *American Institutes for Research* (AIR) comparing the teaching of elementary school mathematics in the United States and Singapore has found that Singapore's textbooks and assessment examinations are more demanding and their teachers more skilled mathematically but that U.S. approaches often put more emphasis on certain important 21st century math skills.

Funded by the U.S. Department of Education, the study *What the United States Can Learn From Singapore's World-Class Mathematics System* (and what Singapore can learn from the United States) identified major differences between the mathematics frameworks, textbooks, assessments, and teacher preparation in both countries.

Singapore is a recognized leader in mathematics achievement. Singaporean students ranked first in the world on the Trends in *International Mathematics and Science Study-2003*, while U.S. students ranked 16th out of the 46 participating nations. Scores for U.S. students were among the lowest of all industrialized countries.

"It is unreasonable to assume that Singaporean students have mathematical abilities inherently superior to those of U.S. students; rather, there must be something about the system that Singapore has developed to teach mathematics that is better than the system we use in the United States. That's why it's important to take a closer look, and see how the U.S can learn and how the U.S can improve," says STEVEN LEINWAND, the lead AIR author. "And in the process, we came across some things Singapore might think about addressing. For example, the U.S. frameworks more often include high-order thinking skills critical to competing in the 21st century, though they are not obviously taught well enough here."

The study also includes initial results from four pilot programs that used the Singapore mathematics textbook in place of their regular textbooks. The pilot programs involved

[1] From the American Institutes for Research website, Washington, D.C. February 7, 2005. See http://www.air.org/news/default.aspx

students in Baltimore, Md., Montgomery County, Md., North Middlesex, Mass., and Paterson, N.J. The study found two pilot sites produced sizeable improvements in student outcomes, but overall the study observed mixed results because "the pilot sites, to varying degrees, encountered problems with teachers who lacked the educational preparation needed." Student mobility also limited prior exposure to the Singapore mathematics curriculum, a serious problem in a curriculum that teaches to mastery and does not repeat content.

Singapore has a centralized educational system, with detailed and consistent implementation procedures that teach topics to mastery at each grade. In order to characterize the decentralized U.S. system, mathematics frameworks in seven states were examined: California, Florida, Maryland, New Jersey, North Carolina, Ohio and Texas.

Findings include:

Singapore Strengths

- Framework: The study indicates there is a correlation between focused frameworks such as those used in Singapore and good test performance. Singapore offers an alternative mathematics framework for lower-performing students that covers all the mathematics topics in the regular framework, but at a slower pace and with greater repetition, and with support from expert teachers.
- Textbooks: Singapore's textbooks build deep understanding of mathematical concepts while traditional U.S. textbooks rarely get beyond definitions and formulas.
- Teaching: Singaporean elementary school teachers are required to demonstrate mathematics skills superior to those of their U.S. counterparts before they begin paid college training to become a teacher. They receive a high level of professional development training (100 hours) each year.
- Assessment: Singapore uses more challenging tests and utilizes a value-added approach that rewards schools for individual student progress over time.

U.S. Strengths

- U.S. Strengths: Although the U.S. mathematics program is weaker than Singapore's in most respects, the U.S. system is stronger than Singapore's in some areas. The U.S. frameworks give greater emphasis than Singapore's to developing important 21st century mathematical skills such as representation, reasoning, making connections, and communication. The frameworks and textbooks also place greater emphasis on applied mathematics, including statistics and probability.

The researchers concluded that the "exploratory results have identified key differences between the U.S. and Singapore mathematics systems. These differences suggest potentially significant reforms that could improve the U.S. mathematics system, but these findings require further validation" from larger scientific studies.

NY: State Changing Math Standards In High Schools[2]

New York is about to end its integrated approach to high school math, and adopt standards more in line with the rest of the country.

The Board of Regents is expected to approve a plan that reorganizes the subject into three, one-year, single-focus courses. Now, New York high schools integrate many different areas of math into each grade.

The new system will require that freshmen take algebra, sophomores study geometry and juniors have algebra II and trigonometry. Schools will have the option of offering pre-calculus or some other course to seniors.

"Students completing these three courses will have a solid background knowledge of mathematics, both from a skills point of view as well as a general understanding," said Dr. ALFRED POSAMENTIER, a member of the committee that drafted the changes and a professor of mathematics at City College.

The state began to re-evaluate its math standards in 2003 when two-thirds of high school students who took the Regents Math A exam failed.

The new standards are considered more challenging because they include probability and statistics, as well as 3-D and transformational geometry.

The Real Scandal in American School Mathematics[3]

Anthony Ralston

It is a scandal that so little attention has been paid to attracting better-qualified math teachers to American schools.

Results from the most recent study of the *Program for International Student Assessment*, or PISA, have highlighted once again the continuing failures of American school mathematics education. ("Poor Math Scores on World Stage Trouble U.S.," Jan. 5, 2005.) These failures have been the subject of a long-running controversy, the so-called *Math Wars*, between research mathematicians and mathematics educators. The debate has centered mainly on matters of curriculum and how or whether technology should be used in math education. By far the most important issue, however, the quality of the nation's cadre of K-12 mathematics teachers, is seldom mentioned.

[2] The New York Times, Tuesday, March 15, 2005

[3] From Education Week, April 27, 2005, Voume 24, Issue 33, p. 35. Anthony Ralston is a professor emeritus of computer science and mathematics at the State University of New York at Buffalo. He lives in London, England.

Recently, a member in good standing of the *Anti-Calculator Brigade* told me the following story. While giving a review course at a private high school for the math section of the SAT, he asked the students how to express 5 + 9/100 + 3/10000 as a decimal.

Every student assembled started punching numbers into a calculator.

It is a cautionary tale, to be sure. But it says less about the use or misuse of calculators than it does about the math teachers who allow students to develop such appalling habits. In fact, all the arguments in recent years about curricula and calculators are virtually irrelevant when compared with the single greatest challenge facing American school mathematics: how to do something about the steady decline over the past half-century of the intellectual abilities of those who teach math in our schools.

Both mathematicians and math educators have urged improving the preservice education of math teachers and providing them with more and better in-service programs to upgrade their skills and knowledge. But practically nothing has been said about the quality of those entering teacher education programs in mathematics.

Why not? One reason may be that mathematicians, who seem quite happy to disparage collegiate-level mathematics educators, don't want to be seen as teacher bashers. This would smack of elitism. It also might seem to be overkill, since more than enough politicians, parents, and others are ready to criticize school teachers.

Nor do I wish to be accused of teacher-bashing. There are many excellent secondary school mathematics teachers, and many elementary school teachers more than capable of teaching just about any mathematics curriculum they are given. Yet if there has been and continues to be a decline in the quality of entrants into school mathematics teaching, this needs to be said. Until such a trend is recognized, arrested, and reversed, nothing else we do about math education will make much difference.

That the quality of people going into teaching has been declining for decades is hardly an original thought. A recent book by former teachers VIVIEN TROEN and KATHERINE BOLES, *Who's Teaching Your Children? Why the Teacher Crisis Is Worse Than You Think and What Can Be Done About It* (Yale University Press, 2003), claims that the number of good classroom teachers is in perilous decline and will continue to worsen. And what is true of classroom teachers generally will be true in spades for mathematics teachers, since the intellectual demands of teaching math are greater than those for almost any other school subject.

But how do we know that this decline in the quality of math teachers actually has taken place? There is some direct evidence: the high number of uncredentialed secondary school mathematics teachersâ¤"indeed, the number who have neither a major nor a minor in mathematics; the poor grasp of basic arithmetic by the above average elementary school teachers studied by Liping Ma in *Knowing and Teaching Elementary Mathematics* (Lawrence Erlbaum, 1999); various studies showing widespread *math phobia*

among elementary school teachers. And then, of course, there is the anecdotal evidence, such as my colleague's calculator story.

(Sidebar: Indeed, the surprising fact is not that the United States has far fewer mathematically competent teachers than it needs, but that it has as many competent ones as it does.)

Quite aside from the evidence, however, there is an irrefutable logical argument to be made that the intellectual level of American school mathematics teachers must have been declining for the past half-century or longer. We know that historically the great majority of schoolteachers in this country, particularly those in the elementary grades, have been women. Until World War II, the only professions generally open to women were teaching and nursing. Since that war, however, all the hitherto male-dominated professions have gradually become accessible to women, some completely, others less so. These other professions are better paid than teaching many are much better paid and exert a powerful career pull for exceptionally talented women who once might have been teachers.

Moreover, during this same period, American schools, particularly those in urban areas, but not only there have steadily become more unpleasant, less safe, and more stressful workplaces. Some people, of course, embrace teaching as a career because of the sheer love of it. But surely not all that many, now or ever. So even though many more women are in the work force than were 50 years ago, it must be the case that fewer and fewer of the best and the brightest go into school teaching, particularly mathematics teaching. So many opportunities exist outside of teaching for the mathematically proficient. Indeed, the surprising fact is not that the United States has far fewer mathematically competent teachers than it needs, but that it has as many competent ones as it does.

Other than a wish to refrain from teacher-bashing, there may be another reason mathematicians generally stay away from this issue: They despair of being able to have an impact on the problem. They have no special expertise here, as they think they do on such matters as curriculum and teacher training.

I share this despair. Attracting the needed numbers of mathematically competent teachers to American schools will not happen in my lifetime, nor in the lifetimes of most of those who read this essay. The language in the federal *No Child Left Behind Act* about having a highly qualified teacher in every classroom is pure cant, because there are no programs in that law that might attract those qualified teachers to American schools. The opposite is true, in fact, because the testing regimen in the law is sure to dissuade people from taking up teaching as a profession. And the increasing use of direct instruction must be anathema to anyone who really wants to help kids learn mathematics.

It is a scandal that so little attention has been paid to attracting better-qualified math teachers to American schools. What can be done?

Instead of all the time and energy spent on arguing about curriculum and related matters, mathematicians and mathematics educators should devote their energies to making the case that those we attract to elementary and secondary mathematics teaching need to be as intellectually able as those attracted to law, medicine, and, yes, the academic world. This means supporting higher salaries and better working conditions for all teachers, in the forums where mathematicians and mathematics educators have some influence: the national academies, the *National Science Foundation*, and whatever other bodies can be influenced in Washington and the state capitals.

This will be a long, hard slog. But on the eventual success of such efforts by mathematicians and math educators (but, of course, not only by them), the future of American education will depend. The No Child Left Behind Act is at least correct in its assertion that high-quality teachers are needed in all classrooms. Without them, future generations will be as mathematically impoverished as the current generation already is.

NCTM has two new position statements:

Closing the Achievement Gap

http://www.nctm.org/about/position_statements/position_achievementgap.htm

and *Computation, Calculators, and Common Sense*

http://www.nctm.org/about/position_statements/computation.htm

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International Newsletter on the Teaching and Learning of Mathematical Proof

The latest issue is now available on-line at the following site:

http://www.lettredelapreuve.it